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# Development of Double Gear Fuel Pump for Heat Management Improvement

*A unique double gear fuel pump system with operation mode switching capability for aircraft engines was developed to solve the heat management problem of current high efficiency turbofan engines and improve specific fuel consumption (SFC). Mode switching from parallel operations to series operations was found to reduce the discharge flow and pump work to nearly half. This resulted in the reduction of the rise in fuel temperature due to the fuel recirculation at the high altitude low Mach number flight condition. Air cooled oil cooler (ACOC) is usually required for sufficient oil cooling at descent or approach flight conditions. Since fuel consumption at those conditions is not very high, most of the gear pump discharge fuel flow proportional to the engine speed is returned to the fuel pump inlet resulting in significant heating. The ACOC that provides additional cooling capability degrades SFC due not only to the increased weight but also to the wasted fan discharge air. By reducing fuel temperature rise at the pump at those flight conditions, the necessity of ACOC may be eliminated. Further, it is shown that a reduction by half of the double gear pump weight can be achieved by increasing pump speed twice without incurring a durability penalty. Extensive tests showed sufficient steady state pump performance, switching characteristics, and durability. [DOI: 10.1115/1.2833492]*

## Introduction

Heat management of fuel and oil systems for turbofans has become a serious problem in current high bypass turbofans with higher fuel efficiency. This is because there is less fuel flow available for oil cooling. The oil that absorbs heat generated at the engine gearbox, bearings, etc., is cooled by heat exchange with fuel at the fuel cooled oil cooler (FCOC). When the FCOC does not have sufficient cooling capability, the air cooled oil cooler (ACOC) is used using the fan discharge air as a coolant. The fan air use for oil cooling deteriorates specific fuel consumption (SFC) since the air is released overboard without producing any thrust. This effect is more significant in high altitude low speed flight condition rather than sea level static (SLS) since fuel consumption of turbofans at the same rotor speed is about proportional to engine inlet pressure. The heat generated at the gearbox, bearings, etc., however, depend on flight conditions but mostly increases with rotor speed.

Fuel temperature increases even in the fuel system. Typical turbofans use gear pumps as the high pressure fuel pump. Since the pump is driven by the engine rotor via the gearbox, the discharge flow of the gear pump is proportional to the engine rotor speed. At high altitude flight conditions, most of the gear pump discharge flow is returned to the pump inlet. Due to the fuel recirculation, the fuel temperature rises exaggerating the heat management problem. Too high fuel temperature tends to cause problems such as fuel nozzle coking.

Several approaches are proposed to reduce the fuel temperature rise due to the recirculation. These approaches include (1) use of a variable capacity pump [1,2], (2) control of the pump speed by means of an electronic motor [1-5] and (3) return of extra fuel to the aircraft tank. The first and second approaches intend to match the pump discharge flow to the actual engine fuel demand. The third approach uses the large aircraft tank capacity to reduce the fuel temperature rise.

In this paper, we propose a unique fuel pump system using a pump with a double gear element as the high pressure pump. This

can reduce the discharge flow to one-half by switching the operation mode of the double gear pump from parallel to series. Another important feature of the proposed system is the reduction of pump weight to about one-half, without any impact on pump durability, by the increase of pump speed.

A fuel system study for an advanced turbofan was conducted. Based on the study's specification, two double gear pumps were designed and manufactured. Bench tests show that the proposed system with the double gear pump met the performance conditions as well as the durability requirements for the advanced turbofan.

## Fuel System and Oil System

A typical diagram of the fuel and oil system for turbofans is shown in Fig. 1. Fuel supplied from the aircraft is pressurized by the low pressure (LP) and high pressure (HP) fuel pumps. The fuel metering unit meters the fuel according to commands from the full authority digital electronic control (FADEC). Extra fuel is returned to the HP pump inlet. After heat exchange with oil at the FCOC, the metered fuel is injected into the engine combustor through the fuel injector.

The purpose of the oil system is to lubricate and cool the bearings and the gearbox as well as cool the sump walls. The oil pump pressurizes oil from the oil tank. The oil is delivered to the sumps and the gearbox where it drains by gravity to the scavenge system and returned to the tank by the scavenge pump. On the way, heat from the oil is transferred to the fuel or fan discharge air through FCOC and ACOC, respectively. Since this is a closed system, the oil temperature will increase above the limit if the absorbed heat is not transferred by the heat exchangers.

## Fuel Temperature Rise Due to Recirculation

Typical fuel systems use a centrifugal pump as the LP pump to enhance suction capability in the case of aircraft boost pump failure and to protect the pump element from cavitation, and a gear pump as the HP pump. Pump sizing depends on pump characteristics and engine demand. In the case of a positive displacement pump such as a gear pump driven by a gearbox, the pump size is determined as the maximum of fuel demand per pump speed

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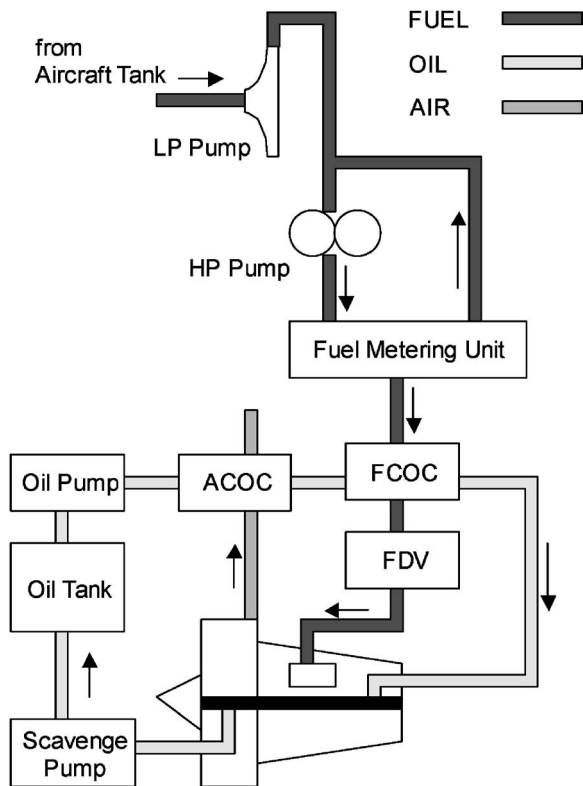


Fig. 1 Typical fuel and lubrication oil system for turbofan engines

throughout the flight conditions and the engine operations. The fuel demand is computed by summing steady state and acceleration fuel flow, actuator actuation flow, and deterioration margin.

The engine fuel demand at the same rotor speed is roughly proportional to the engine inlet pressure. At high altitude low Mach number flight conditions, the fuel demand significantly decreases from that of the SLS condition. The gear pump discharges fuel proportional to the engine speed. The extra fuel is bypassed and returned to the gear pump inlet where the energy provided by the pump is dissipated into heat. The fuel temperature rise due to the recirculation is determined by the difference between the pump work and the heat carried away by the fuel to the combustor. The pump work is given by

$$L_p = \text{Tr}q_p \omega_p = \frac{1}{\eta_p} Q_p P_o$$

Since the dissipated work is  $L_p - Q_b P_o$ , the fuel temperature rise due to the recirculation is given by

$$\Delta T_{fp} = \frac{L_p - Q_b P_o}{C_{pf} \rho_f Q_b} = \frac{P_o}{C_{pf} \rho_f} \left( \frac{Q_p}{\eta_p Q_b} - 1 \right)$$

As the recirculation ratio increases, it is clear that fuel temperature rises rapidly. The rise in fuel temperature is proportional to the pump discharge pressure, which varies with the flight condition and engine rating. The maximum fuel temperature rise at the pump is often encountered at the approach flight condition. A typical example shows that fuel consumption is only from 1/40 to 1/20 of the pump discharge flow. Also, the fuel temperature rise (due to the fuel recirculation) reaches about 50°C for advanced high bypass turbofans.

Since the fuel is used to cool the engine oil (described in the following section), the high fuel temperature rise at the pump may cause serious heat management problems. Additional equipment needed to provide sufficient oil cooling worsens engine SFC.

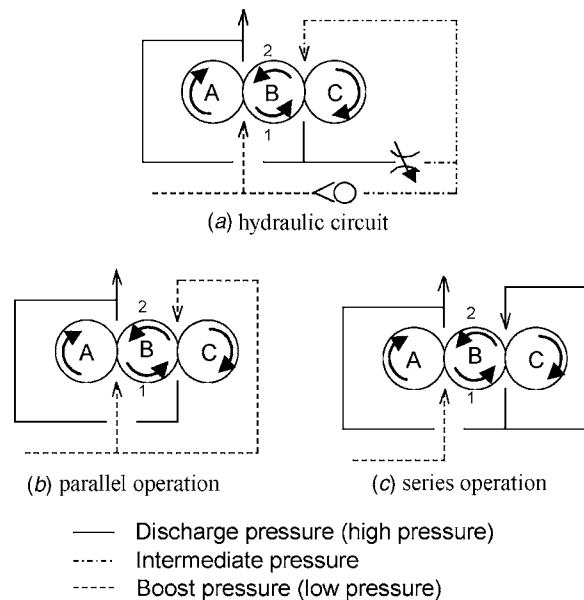


Fig. 2 Hydraulic circuit and operation modes

Usually, ACOC is used to provide extra oil cooling. If the heat generation at the pump is reduced to less than half, it is possible to eliminate the ACOC, which not only adds engine weight but also wastes the fan discharge air. Motivated by this, we propose a unique fuel pump system based on well-developed gear pump technology. The main feature of the system is the use of a pump with a double gear element. By switching the pump operation mode from parallel to series so that the recirculation fuel temperature rise can be reduced, at a high altitude low Mach number flight (where less fuel is required compared with a low altitude high Mach number condition), the pump discharge flow is reduced to half.

## Double Gear Pump System Design

**Hydraulic Circuit.** The proposed system has a simple hydraulic circuit that consists of a double gear pump, a check valve, and a variable orifice, as shown in Fig. 2(a). The system has two operation modes. At the parallel operation mode, the variable orifice is closed. Fuel from the aircraft tank is directly supplied to Elements A and B. The fuel is also supplied to Elements B and C past the check valve with low cracking pressure. The fuel flows to the inside of the pump along with arrows. Thus, the pump discharge flow is the sum of Elements A and B1 and B2 and C, as shown in Fig. 2(b).

As the variable orifice opens partially, part of the discharge flow of Elements B1 and C is supplied to the inlet of Elements B and C. This results in the decrease of total pump discharge flow. As the opening of the variable orifice increases, the flow to the inlet of Elements B and C increases and the pressure difference across Elements B and C decreases. The check valve prevents the flow from returning to the supply. When the variable orifice is completely open, the work of the Elements B2 and C is almost zero since the pressure at the inlet of Elements B and C is approximately equal to the pump discharge pressure. Hence, in the series operation where the variable orifice is open, both the discharge flow and work become about half of the parallel mode.

**The Internal Leakage of Double Gear Pump Affect on Volumetric Efficiency.** In order to minimize fuel leakage between the bearing and the gear, a common practice is to use a floating bearing. Axial pressure loading is applied to the floating bearing by venting pump discharge pressure to the top of the floating bearing. Although the inlet pressure of Elements B and C changes from

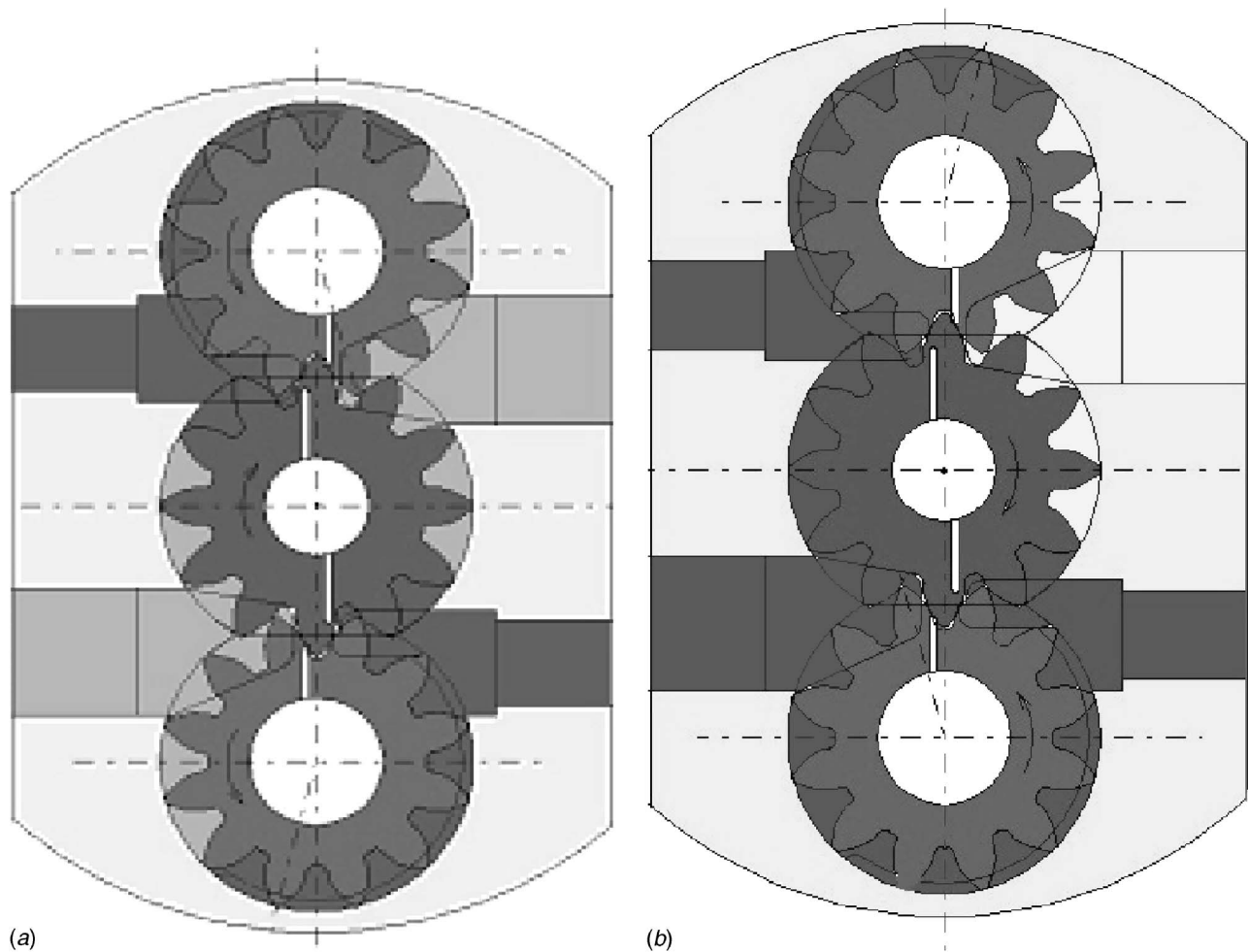


Fig. 3 Pressure distribution on floating bearings top: (a) parallel operation; (b) series operation

low to high according to the operation mode change, it is possible to design the bearing so the direction of the axial loading force is always toward the gear and the magnitude of change does not have a significant impact on the pump's performance and life. The dark area in Figs. 3(a) and 3(b) shows the HP area.

Nitriding steel or tool steel is used as the gear element material and aluminum as the case. The coefficient of linear expansion of the aluminum is much larger than that of the gear element material. In order to prevent rubbing even at low temperature operations ( $-54^{\circ}\text{C}$ ), sufficient clearance between the gear tooth tip and the case must be ensured. To minimize the tip leakage from the discharge to the inlet, a common practice is to use a radial pressure loading concept, which hydraulically loads the gear/bearing assembly toward inlet. However, this concept cannot be applied to Element B at the parallel operation, as shown in Fig. 4(a). The tip leakage might cause considerable volumetric efficiency decrease under ordinary and high temperature operations. No radial force is applied to Element C at the series operation, as shown in Fig. 4(b). Since the element C does not contribute to the discharge flow, this does not matter.

A detailed analysis is done to estimate the tip leakage. By assuming that the flow in the tip clearance is modeled as Hagen-Poiseuille flow shown in Fig. 5, the velocity profile is expressed as [6]

$$u = \frac{\Delta P_o}{2\mu_f z_0 t} (\delta - y)y - \left(1 - \frac{y}{\delta}\right)U \quad (1)$$

The first term of the right hand side represents parabolic velocity profile due to the discharge to inlet pressure difference.  $z_0$  is the

number of teeth that lie between discharge and inlet pressure. The pressure difference across each gear is approximately  $\Delta P_o/z_0$ . The second term is linear velocity profile caused by tip speed,  $U$ . Integrating the velocity profile from the tooth tip to the casing gives the leakage flow,  $Q_l$ .

$$Q_l = 2b \int_0^{\delta} u dy = \frac{b\Delta P_o}{6z_0 t \mu_f} \delta^3 - U\delta b \quad (2)$$

**Pump Weight and Size Reduction.** Given discharge flow and pressure, it is possible to design various gear pumps. Since so called bearing  $PV$  value has a significant effect on the pump durability, the maximum allowable  $PV$  value is specified. Under those conditions, relations between module, number of teeth, and speed are derived. Then, the geometry of the gear and the bearing are determined and the weights are computed.

Theoretical gear pump discharge flow is given by

$$Q_{th} = \frac{\pi b m^2}{2} \left\{ (z+2)^2 - z^2 - \frac{\pi^3}{3} \cos^2 \alpha_0 \right\} n \quad (3)$$

The bearing load that consists of the hydraulic radial force and the reaction force to drive the gear is approximated by

$$F_b = 0.8\Delta P_o D_2 b \quad (4)$$

Hence, the bearing mean pressure is

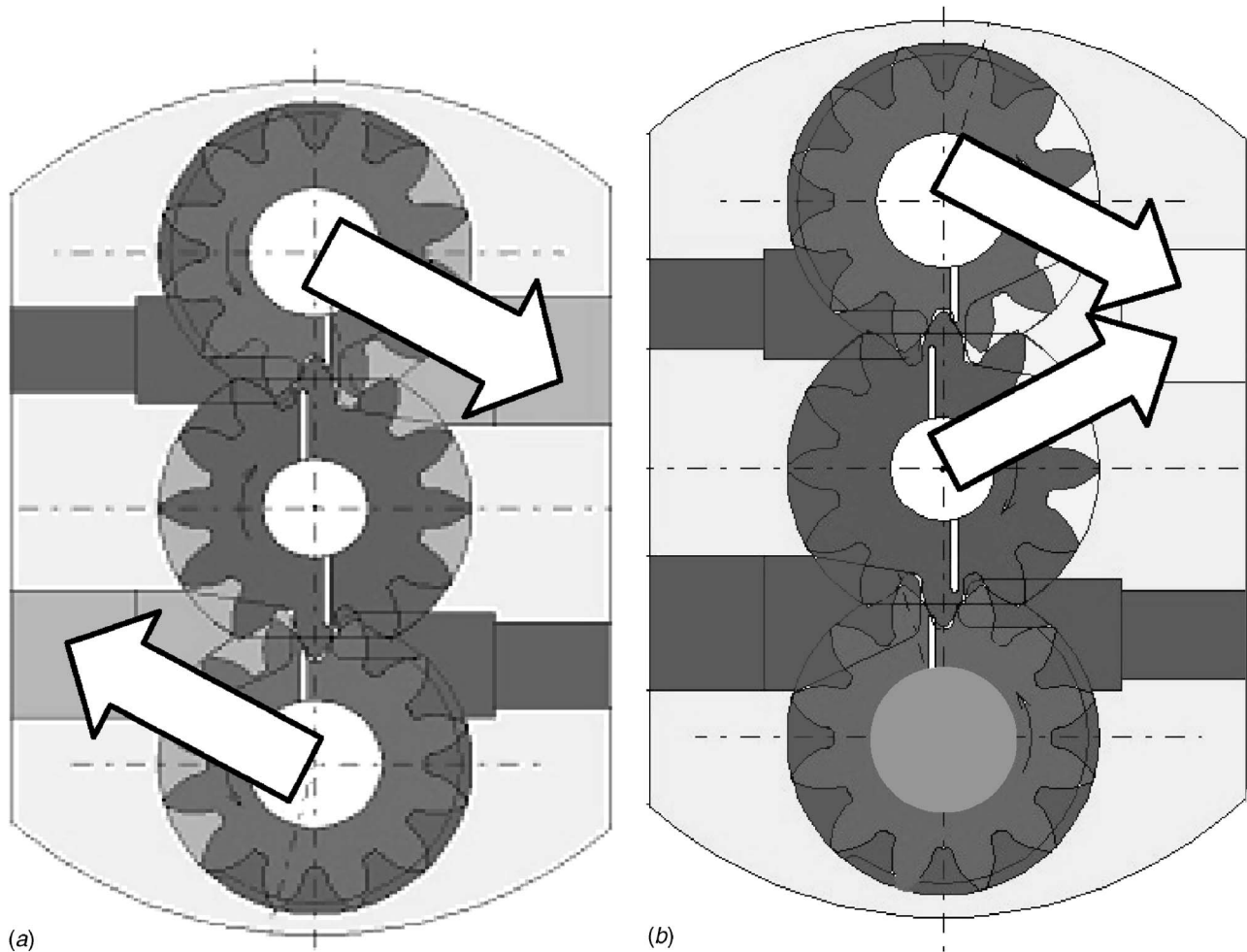


Fig. 4 Radial force direction: (a) parallel operation; (b) series operation

$$P_b = \frac{0.4\Delta P_o D_2 b}{D_b L_b} \quad (5)$$

The circumferential speed of the bearing is

$$V_b = \pi D_b n \quad (6)$$

Eliminating  $n$  and  $b$  from above three equations gives the following relation between the module and the number of teeth.

$$m^2 = \frac{4(z+2)}{20 \frac{L_b}{D_b} (z-5)(z+1 - (\pi^2/12)\cos^2 \alpha_0)} \frac{Q_{th} \Delta P_o}{P_b V_b} \quad (7)$$

From Eq. (3),  $b$  times  $n$  is given by

$$bn = \frac{Q_{th}}{2\pi m^2 (z+1 - (\pi^2/12)\cos^2 \alpha_0)} \quad (8)$$

Note that the following are assumed to derive the above equations. (a) Ratio of the bearing diameter and length is constant.

$$\frac{L_b}{D_b} = \text{const}$$

(b) Diameter of the bearing is given by

$$D_b = m(z-5)$$

Figure 6 shows the weight versus speed of double gear pumps and conventional single gear pumps computed by using the above equations. The discharge pressure and flow are 6.895 MPa and 0.118 m<sup>3</sup>/min, respectively. The bearing PV value is 26.97 MPa m/s. The number of teeth is used as the parameter. Both ends of each curve are limited by maximum  $P_b$  (=30 MPa) or maximum  $V_b$  (=15 m/s). The double gear pump with increased pump speed may realize substantial weight reduction.

The bearing  $P_b$  value of the double gear pump is half of the conventional pump at the same pump speed. Since the double gear

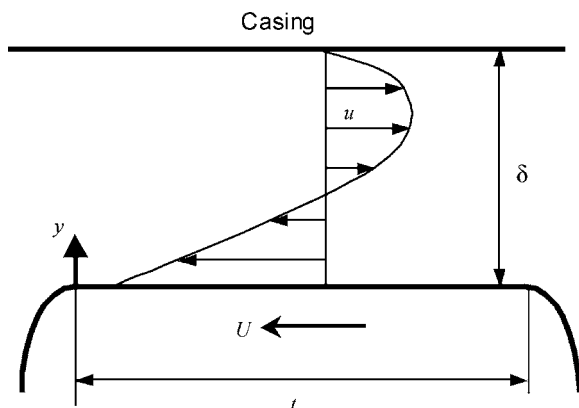


Fig. 5 Flow inside tip clearance

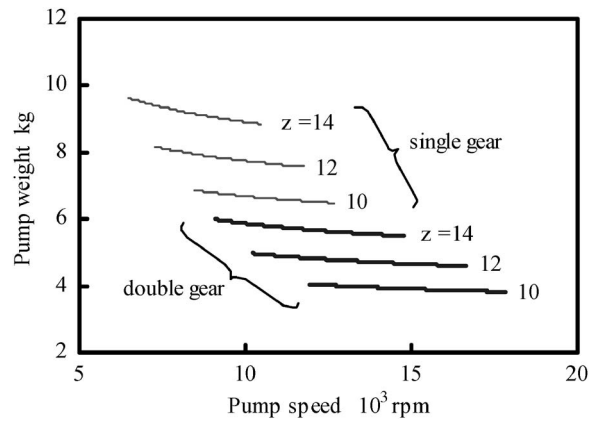


Fig. 6 Pump weight versus pump speed

pump consists of two gear elements with half tooth width. So at the same  $PV$  (i.e., the product of  $P_b$  and  $V_b$ ) value of bearing, the double gear pump speed can be doubled. Then the discharged flow rate can be doubled. So the pump size can be reduced. Hence, it is expected that the proposed double gear pump system with 15,000 rpm and operation mode switching capability may reduce not only the pump work but also the pump weight to half of the conventional single gear pump with 7500 rpm.

### Double Gear Pump

As previously discussed, the volumetric efficiency might not be so high at the parallel operation. An important design objective is to minimize the volumetric efficiency decline as well as to maximize weight reduction. Our design approaches are as follows: (a) to increase  $PV$  value, (b) to reduce the number of teeth, and (c) to increase pump speed

Higher  $PV$  value obviously makes the pump size smaller. Number of teeth reduction results in lighter weight, as shown in Fig. 6. The pump speed seems to have little effect on the weight. However, increasing pump speed makes the gear width smaller (see Eq. (8)) resulting in leakage reduction. The smaller diameter makes the tip clearance smaller, which also results in leakage reduction.

To validate our design approaches, we designed, manufactured, and tested two double gear pumps. Main features are shown in Table 1.

Cross-section diagram of Type H is shown in Fig. 7, and a three-dimensional external and cutaway view is shown in Fig. 8. The pump includes the mode switching valve (variable orifice) actuated by a torque motor, the check valve, and the necessary

Table 1 Double gear pump specifications

	Type H	Type C
Theoretical discharge flow rate in parallel operation ( $m^3/min$ )	0.125	0.140
Theoretical discharge flow rate in series operation ( $m^3/min$ )	0.063	0.070
Speed (rpm)	15,000	7500
Number of teeth	10	12
Gear width (mm)	10.3	13.4
Pitch circle diameter (mm)	25	36
Tooth tip diameter (mm)	30	42
Bearing diameter (mm)	13	25
$PV$ value (MPa m/s)	37.27	13.83
Weight (kg)	4.5	10

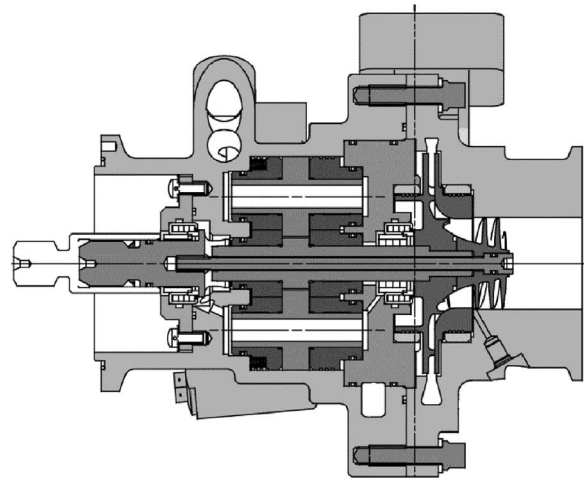


Fig. 7 Cross-section diagram of Type H

passages. The pump also has a centrifugal boost stage for cavitation prevention. Main components of Type H and Type C pumps are shown in Fig. 9.

The tip leakage of Type H may be smaller than Type C because of greater tooth tip speed, smaller clearance (due to smaller diameter), and smaller gear width. Equation (2) gives the tip leakage flow at  $30^\circ C$ . It is from 0.1% to 1.4% of discharge flow for Type H and from 1.1% to 5.5% for Type C. The volumetric efficiency is predicted to be better for Type H than Type C.

As predicted, it was confirmed that during the parallel operation the volumetric efficiency decreases due to the leakage from gear tip. However, it is also supposed that the volumetric efficiency during parallel operation approximately can be increased up to the volumetric efficiency of the conventional pump, because the leakage from gear tip can be reduced up to 1.1% of the pump discharged flow rate if the gear diameter and the gear tip speed are

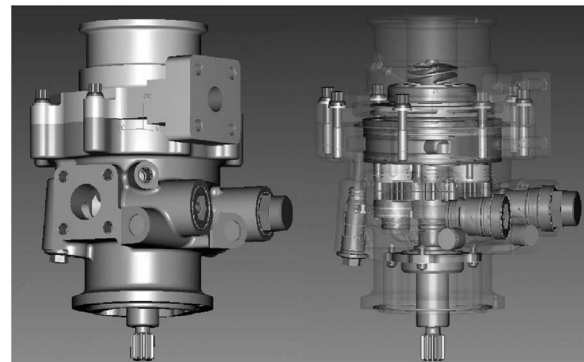


Fig. 8 3D external and cutaway view of Type H

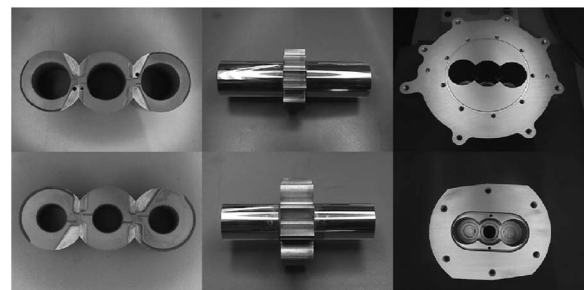


Fig. 9 Main components of Type C (top) and Type H (bottom)

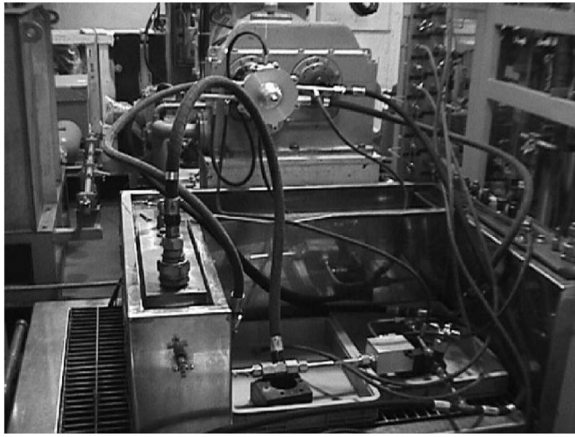


Fig. 10 Bench test setup

designed such as the Type H. In general, the volumetric efficiency of the conventional pump is from 9% to 95% due to the leakages and the cooling flow at the bearing. It is expected that the decrease volumetric efficiency of the parallel operation will increase to that

of the conventional pump when the bearing cooling flow rate is properly designed as mentioned previously.

### Bench Test Results

**Steady State Performances.** Bench tests were carried out to demonstrate the double gear pump performance (Fig. 10). The block diagram of the bench test setup is shown in Fig. 11. Steady state  $P$ - $Q$  performances of Type H (15,000 rpm) and Type C (7500 rpm) are shown in Figs. 12(a) and 12(b), respectively.

The volumetric efficiencies of Types H and C are shown in Fig. 13. Contrary to the prediction, the volumetric efficiency of Type H is lower than Type C.

Another observation is that the volumetric efficiencies of Type H decrease similarly with the increase of discharge pressure for both parallel and series operations. This was caused by large amount of bearing lubrication fuel flow. Since the  $PV$  value of Type H is high, necessary lubrication flow for the bearing was conservatively estimated to prevent bearing failure. The lubrication flow, we believe, can be substantially reduced by further analysis and tests. Thus, the efficiency decrease does not change with the operation modes. The volumetric efficiency tendency of Type C is different from Type H. Since the  $PV$  value of Type C is much lower, the bearing lubrication fuel flow can be reduced. In

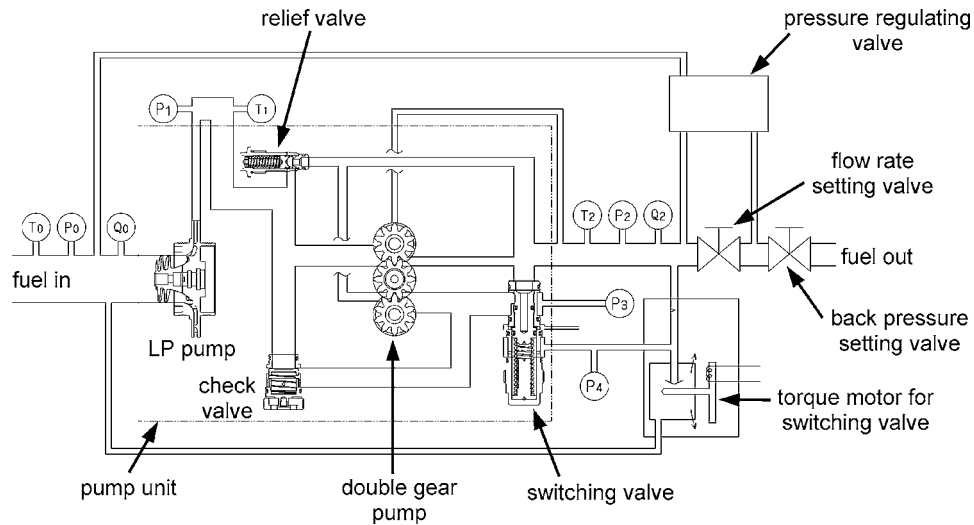


Fig. 11 Block diagram of bench test setup

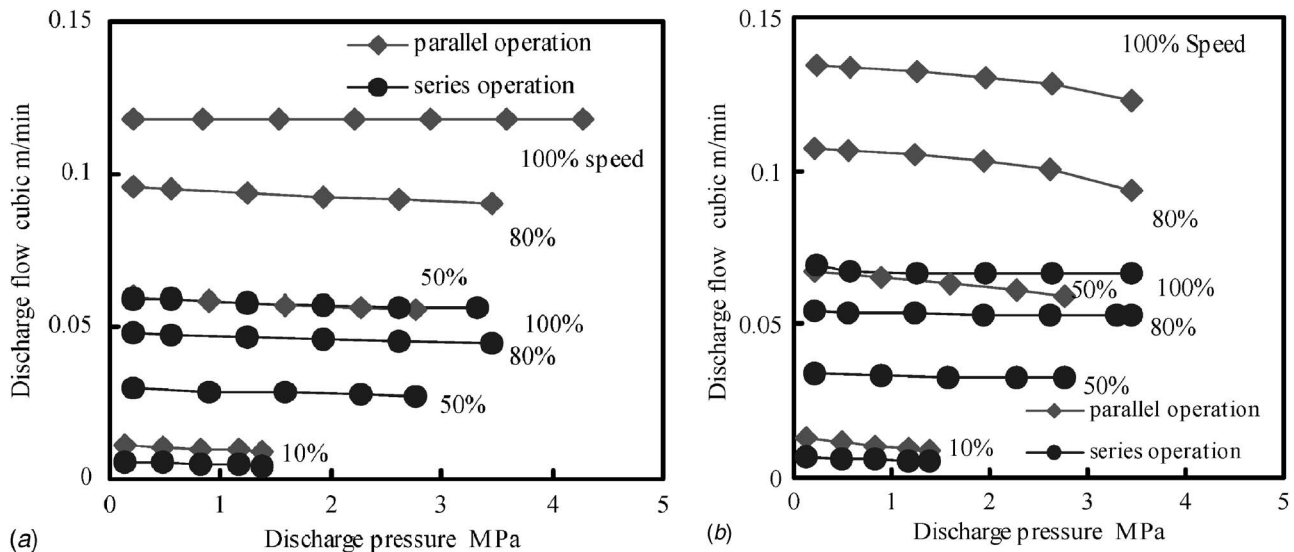


Fig. 12  $P$ - $Q$  characteristics: (a) Type H; (b) Type C

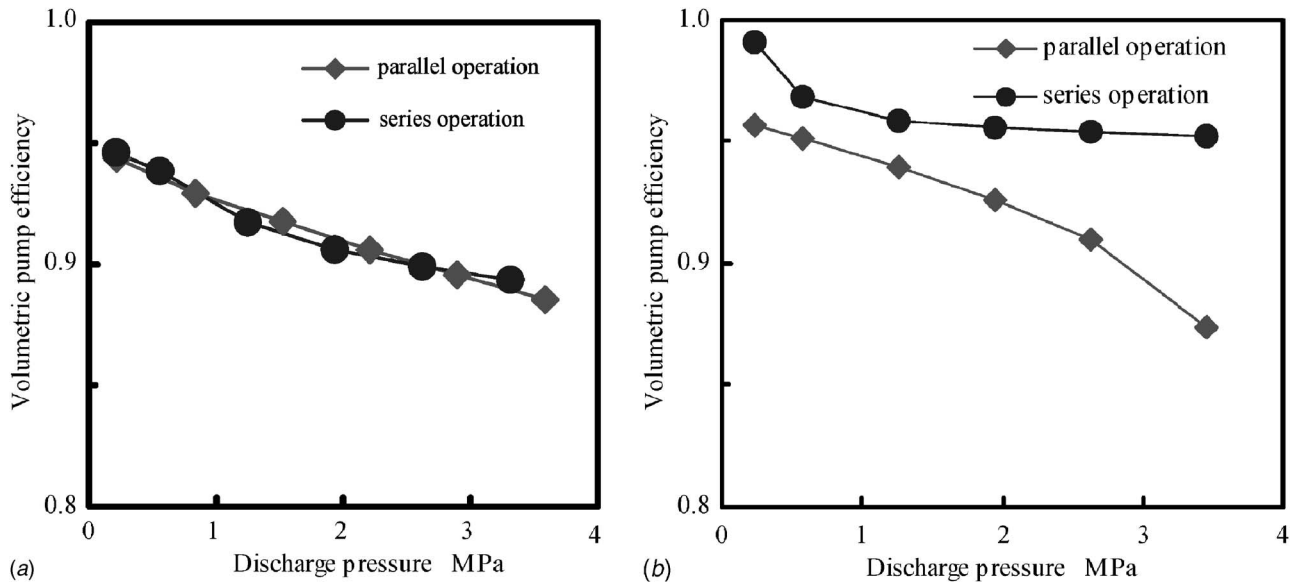


Fig. 13 Double gear pump volumetric efficiency: (a) Type H; (b) Type C

the case of parallel operation, the volumetric efficiency is observed to decrease with the discharge pressure increase. However, significant efficiency decrease is not observed in the series operation. The tendency of Type C concurs with the tip leakage prediction mentioned in the previous section.

**Transition Behavior During Mode Switch.** When the operation mode is switched from parallel to series, the discharge flow reduces to about half. During the transition phase from parallel to series and series to parallel, the metered fuel flow must not be affected. The metered fuel flow fluctuation not only depends on the pump mode change but also the fuel metering system performance. A block diagram of typical fuel metering systems is shown in Fig. 14.

The fuel metering technique is based on volumetric fuel flow proportional to the port area of the metering valve and the root of pressure difference between discharge and inlet of the metering valve. The metering port area is controlled by FADEC and the difference pressure is kept constant by a pressure regulating valve. If the pump discharge flow decreases, the pressure regulating valve reduces the bypass flow port area to reduce the recirculation flow to the pump inlet. It is clear that the slower pump mode transition is preferable.

Figure 15 shows the qualitative behavior of discharge flow and intermediate pressure during mode transition. The intermediate pressure means the inlet pressure of Elements B and C. When the variable orifice is partially open, the discharge flow decreases while the intermediate pressure remains constant since the fuel is

supplied past the check valve with little pressure drop. With further opening of the orifice, the intermediate pressure starts to increase and the check valve is closed. After the check valve closure, the intermediate pressure is determined by pressure drop across the variable orifice. The pump work of Elements B and C decreases as the intermediate pressure increases. When the intermediate pressure reaches the pump discharge pressure, the total pump work reduces to half of the parallel operation.

**Switching Durability Test.** One of the mode switching test results is shown in Fig. 16 in which switching was done manually with a needle valve. The operation mode was switched from parallel to series then back to parallel. The behavior of the discharge flow and the intermediate pressure were as expected. The discharge pressure fluctuation was within 5% of the discharge flow.

Figure 17 shows the  $P-Q$  characteristics before and after the switching durability test. One cycle consists of switching from the parallel to series operation then back to parallel. After 1000 cycle test, no significant performance change was observed.

**Pump Work and Fuel Temperature Rise Reduction.** Figure 18 shows the normal discharge pressure range during flight. The pump work can be reduced by 38–49% by mode switching from

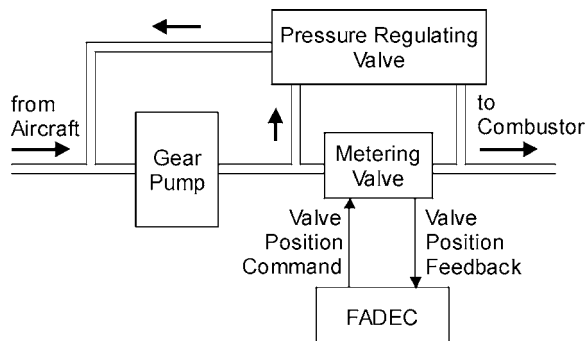


Fig. 14 Typical fuel metering system

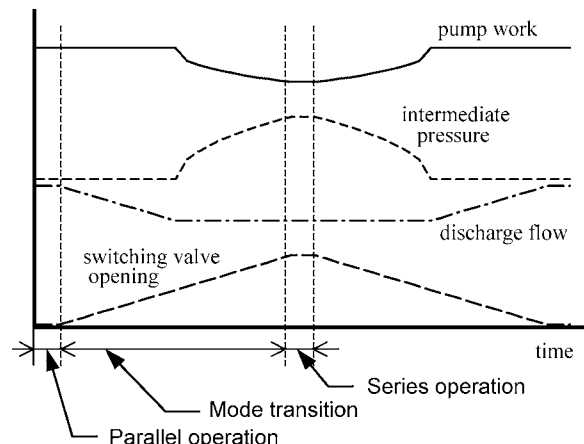


Fig. 15 Mode transition behavior

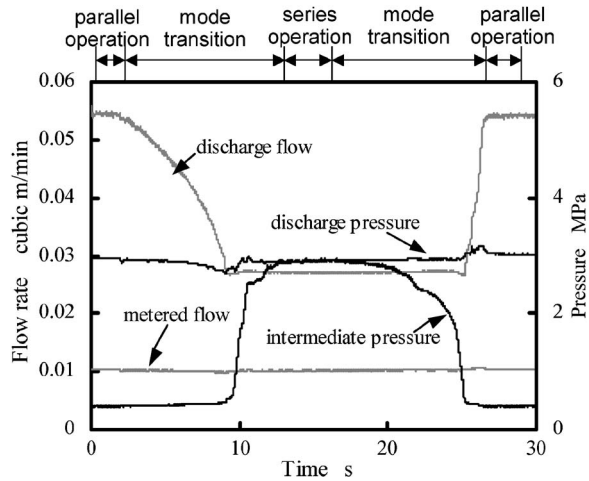


Fig. 16 Mode switching test result

parallel to series. Lower pump work reduction at lower discharge pressure range means that that mechanical loss does not decrease with the discharge pressure.

### Heat Management Sample Computation

In order to demonstrate effectiveness of the proposed pump system, a representative heat management computation is carried out. The engine is assumed to be an advanced high bypass turbofan. By running the engine performance program, the needed engine variables such as fuel flow, engine speed, and combustion pressure are computed over an entire flight envelope.

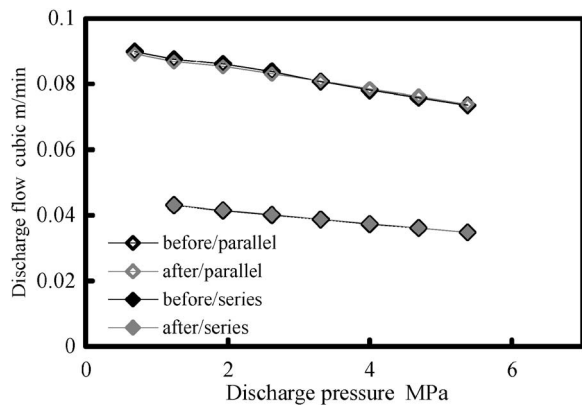


Fig. 17 Switching durability

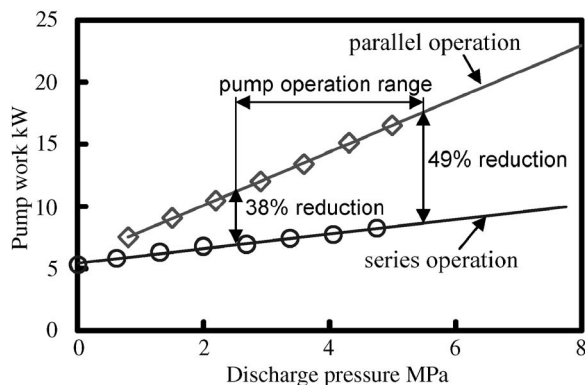


Fig. 18 Pump work reduction of Type H

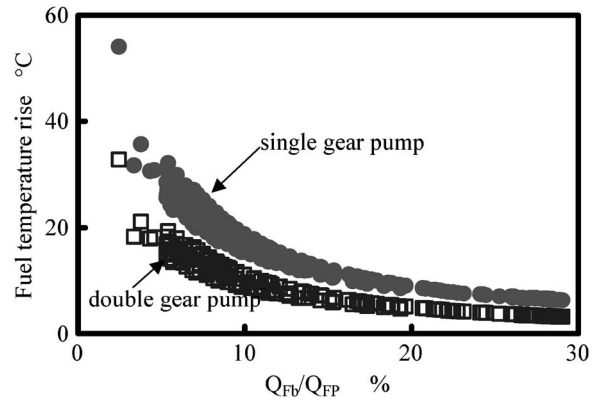


Fig. 19 Fuel temperature rise

The heat absorbed by the oil includes the heat from the bearings, the seals, the sump walls, the pressurizing air, and the gear trains in the gearbox. The heat generated by oil stirring, so called disk friction effect, may be present. An empirical prediction model that is a function of the engine speed is used as the engine heat generation model.

Hot tank type oil system is assumed. The oil temperature in the oil tank is the average of the oil temperatures scavenged from the engine sumps by the scavenge pumps. The oil is cooled by FCOC (shell and tube type) before being supplied to the engine. The heat exchange amount is the function of the flow rates and the temperature differences, which will depend on the flight condition and engine rating.

The fuel pump discharge flow is calculated given the engine speed. The pump discharge pressure is computed by summing the combustion pressure, and various pressure drops such as fuel injector, flow divider, minimum pressure rising valve, and fuel metering valve. Since the burn fuel is given by an engine performance table, the fuel recirculation ratio and the fuel temperature rise are computed. After absorbing the oil heat at the FCOC, the burn fuel is supplied to the combustor.

Figure 19 shows the fuel temperature rise as the function of burn fuel ratio over pump discharge flow. The flight envelope is from sea level up to 38 kft. The engine rating is idle to maximum power. The double gear pump makes the fuel temperature rise about half due to the recirculation.

### Conclusions

A unique fuel system consisting of the double gear pump, the switching valve, and the check valve was proposed in order to solve the heat management problem of advanced high bypass turbofans. The system may also eliminate the ACOC, which will increase the engine weight and waste the fan air. In summary, by switching the operation mode from parallel to series, the reduced pump discharge flow and pump work result in a significant reduction to half of the fuel temperature rise due to the recirculation. The increase of pump speed to double may reduce the pump weight to half of the conventional gear pump system with no durability penalty. Bench tests demonstrated that the proposed system met the requirements for pump performance, switching characteristics, and switching durability.

### Acknowledgment

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## Nomenclature

$b$	=	tooth width
$C_{pf}$	=	specific heat of fuel
$D_2$	=	tooth tip diameter
$D_b$	=	bearing diameter
$\Delta T_{fp}$	=	fuel temperature rise at pump
$F_b$	=	bearing load
$L_b$	=	bearing length
$L_p$	=	pump work
$m$	=	gear module
$n$	=	pump speed
$P_b$	=	bearing mean pressure
$P_i$	=	inlet pressure of pump
$P_o$	=	pump discharge pressure
$\Delta P_o$	=	pressure rise across the high pressure pump
PV	=	PV limit or PV value of journal bearing (i.e., the product of $P_b$ and $V_b$ )
$Q_b$	=	volumetric burn flow rate
$Q_l$	=	volumetric tip leakage flow rate
$Q_p$	=	volumetric pump discharge flow rate
$Q_{th}$	=	volumetric theoretical pump discharge flow rate
$t$	=	tooth tip thickness
Trq <sub>p</sub>	=	pump torque
$u$	=	flow velocity in tip clearance
$U$	=	tooth tip circumferential speed

$V_b$	=	bearing circumferential speed
$y$	=	radial distance from tooth tip
$z$	=	number of teeth
$z_0$	=	number of teeth between inlet and discharge pressure
$\alpha_0$	=	pressure angle
$\delta$	=	tip clearance
$\eta_p$	=	pump efficiency
$\mu_f$	=	fuel viscosity
$\rho_f$	=	fuel density
$\omega_p$	=	pump angular velocity

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